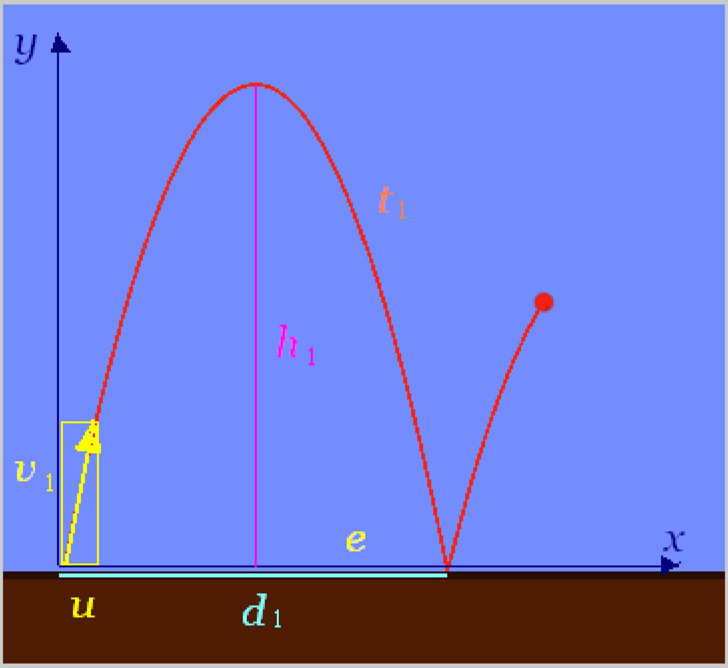
【解答】水平面との無限回衝突

【問題】⇒ 水平面との無限回衝突



(1)

$$0^2 - {v_1}^2 = 2(-g)h_1 \qquad \therefore h_1 = \frac{{v_1}^2}{2g}$$

最高点までの時間が $t_1/2$だから，

$$t_1 = \frac{2v_1}{g}$$

$$\therefore d_1 = ut_1 = \frac{2uv_1}{g}$$

(2)

運動の対称性から，衝突直後と次の衝突直前の速度の y成分の大きさは等しいから，

$$v_n = ev_{n-1} = e^{n-1}v_1$$

$$h_n = e^2 h_{n-1} = e^{2(n-1)}h_1 = e^{2(n-1)} \cdot \frac{{v_1}^2}{2g} \qquad \because h_n = \frac{{v_n}^2}{2g} = \frac{e^2{v_{n-1}}^2}{2g} = e^2 h_{n-1}$$

$$t_n = et_{n-1} = e^{n-1}t_1 = e^{n-1} \cdot \frac{2v_1}{g} \qquad \because t_n = \frac{2v_n}{g} = \frac{2ev_{n-1}}{g} = et_{n-1}$$

$$d_n = ed_{n-1} = e^{n-1}d_1 = e^{n-1} \cdot \frac{2uv_1}{g} \qquad \because d_n = \frac{2uv_n}{g} = \frac{2euv_{n-1}}{g} = ed_{n-1}$$

(3)

求める合計時間は，

$$T = \sum_{n=1}^{\infty} t_n = \sum_{n=1}^{\infty} e^{n-1}t_1 = \frac{t_1}{1-e} = \frac{2v_1}{(1-e)g}$$

また，求める x座標は，

$$X = \sum_{n=1}^{\infty} d_n = \sum_{n=1}^{\infty} e^{n-1} d_1 = \frac{d_1}{1-e} = \frac{2uv_1}{(1-e)g} \qquad \text{or} \quad X = uT = \frac{2uv_1}{(1-e)g}$$